DOCUMENT RESUME

ED 233 074 TM 830 528

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TITLE Methods for Assessing the Effects of Final Demand on Industry and Occupational Employment. Technical Note No. 2.
INSTITUTION Technology and Education Research Inst., Inc., Cambridge, MA.
PUB DATE Apr 83
NOTE 30p.
AVAILABLE FROM Technology and Education Research Institute, 31 Gorham St., Cambridge, MA. 02138 ($3.50).
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Economic Research; Employment Practices; *Evaluation Methods; *Industry; *Input Output Analysis; Labor Market; *Occupations; Productivity; Program Effectiveness
IDENTIFIERS *Demand; Gross National Product

ABSTRACT

There have been developments in interindustry economics which make it useful to reassess the potential for using input-output analysis for study of industry and occupational employment. The basic data sources available for conducting input-output analysis of employment include the Bureau of Economic Analysis in the U.S. Department of Commerce. Notations, definitions, and conventions established are used in developing a number of mathematical expressions of unit terms which are useful in assessing the effect of various patterns of final demand upon employment by occupation and industry. The relevant matrix manipulations are explicated with a simple example of an economy with three industries and three commodities. The final section discusses the application and use of the various expressions developed. (CM)

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Forward

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The present document is the second in the series of Technical Notes.

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Introduction. The potential for using the input-output framework of interindustry economics for the analysis of occupational employment has long been recognized. While most input-output studies treat labor as a homogeneous input to the production process (4), some of the early studies on Leontief's Harvard Economic Research Project were concerned with changing occupational patterns (8). Alfred Conrad, for example, used 1939 and 1947 input-output matrices to determine the effect of technical change on occupational employment (6). More recently Bezdek has provide a useful summary of how interindustry analysis can be used to address manpower questions, including a discussion of the problems encountered when trying to operationalize such studies (1). Since the time that Bezdek wrote, there have been developments in interindustry economics which make it useful to reassess the potential for using input-output analysis for the study of industry and occupational employment. The present study is such an endeavor.

The section to follow describes the basic data sources available for conducting input-output analysis of employment. The notations, definitions, and conventions to be used in the remainder of the paper are then established. These definitions are used to develop a number of mathematical expressions useful in assessing the effect of various patterns of final demand (i.e., the distribution of gross national product by commodities) upon employment by occupation and industry. The matrix manipulations in this section are explicated with the use of a simple example. In a final section there is a discussion of the application and use of the various expressions developed earlier.

The Basic Data. Interindustry economics is concerned with the interrelationships among producing industries as buyers of each others' outputs, as users of such resources as labor and capital, and as sellers to final consumers (5). Interindustry accounts are based upon periodic censuses of all economic sectors and, in the United States, are consistent with the national income and product accounts. Interindustry data, or input-output data, is typically displayed in the form of various kinds of tables. A number of these tables will be illustrated and used later in this paper.

The principal source of data on the input-output structure of the U.S. economy is prepared by the Bureau of Economic Analysis (BEA) in the Department of Commerce. Figure 1 shows the various benchmark studies of the BEA, with brief descriptions of some characteristics of the studies. Since a full benchmark study normally takes five to seven years to complete, the BEA also prepares updated input-output tables for those years subsequent to the latest benchmark study (22). Other sets of input-output data exist in addition to those shown in Figure 1. Leontief and his co-workers constructed tables for 1919, 1929, and 1939 (9). An input-output table for 1947 was constructed by the Bureau of Labor Statistics, with the assistance of the Harvard Economic Research Project. Still other input-output data sets have been prepared for use in various governmental research programs. For example, the Bureau of Labor Statistics has used the basic BEA benchmark data sets to construct input-output tables for use in their Economic Growth Studies (17).

Until 1972 the principal distinctions between the benchmark input-output data sets occurred because different industrial classification systems and different levels of industrial detail were used (see Figure 1). With the 1972 input-output study, however, the BEA made a major change so that U.S. studies

Figure 1: Benchmark Input-Output Studies in the United States[a]

Name	Published Source	Industry Classification	Comments
OBE 1958 Interindustry Relations Study	SCB, Nov. 1964 (7)	85 industries based upon the 1957 SIC	Most secondary products are transferred
OBE 1963 Interindustry Study	SCB, Nov. 1969 (15)	367 industries based upon the 1957 SIC; data also available at the level of 85 industries	Most secondary products are transferred
BEA 1967 Interindustry Study	SCB, Feb. 1974 (16)	367 industries based upon the 1967 SIC; data also available at the level of 85 industries	Most secondary products are transferred
BEA 1972 Interindustry Study	SCB, Feb. 1979 (13)	496 industries based upon the 1972 SIC; data also available at the level of 85 and 367 industries	Secondary products are redefined

[a]Abbreviations:
OBE: U.S. Dept. of Commerce, Office of Business Economics
BEA: U.S. Dept. of Commerce, Bureau of Economic Analysis
SIC: Standard Industrial Classification (11)
SCB: Survey of Current Business

would be compatible with conventions recommended by the United Nations. This change concerns the treatment of secondary products. One consequence of the change is that it is now possible to make the distinction between industry output and commodity output. Some definitions will help in understanding the distinction between the two:

Def. 1: An establishment is an economic unit, generally at a single physical location, where business is conducted or where services or industrial operations are performed (11).

Def. 2: An input-output industry is a grouping of establishments, as classified by the 1972 Standard Industrial Classification (SIC).

Def. 3: An establishment is classified in an industry based upon its principal activity. The output of that principal activity is called the primary output of the establishment.

Def. 4: Any subsidiary output produced by an establishment is called secondary output.

Def. 5: An input-output commodity consists of the characteristic primary product of the corresponding industry, wherever made (12).

The important words in this last definition are "wherever made," for total commodity output is the sum of the output of a given product, regardless of the industry in which it was produced. Total industry output, on the otherhand, is the sum of all output produced by the industry, regardless of the commodity produced. Industry output and commodity output are rarely identical, although if there were no secondary products produced by establishments the output of each industry would be equal to the output of the corresponding commodity.

Figure 2 shows the format for the basic input-output table. This particular table is known as the "use" table. Across a row it shows the use by each industry and the sales to final users (personal consumption, government, investment, and exports) of the output of the commodity named at the beginning of the row (22). Down a column, the use table shows the dollar value of the inputs of commodities and the value-added (sum of compensation of employees, profit-type income, net interest, capital consumption allowances, and indirect business taxes) which are generated in the production of the industry named at the head of the column. The row total is total commodity output. The column total is total industry output.

A second basic input-output table is known as the "make" table. Across the row, a make table shows the dollar value of each commodity produced by the industry named at the beginning of the row. Down a given column, the table shows the dollar value of the given commodity which is produced in each industry. The row total is industry output. The column total is commodity output.

Use and make tables comprise the essential input-output data needed for the analysis of occupational employment. However, input-output data must be used in conjunction with information on the occupational structure of industries. This information is based upon a table of occupational employment by industry,

Figure 1: The Format of an Input-Output "Use" Table[a]

	Intermediate Use							Final Use	Total Use
	Purchasing Industry								
	1	2	.	.	.	.	n	Total	Final Demand	Commodity Output
Commodity 1									
Commodity 2									
.									
.				U				z	y	x^c
.									
.									
Commodity n									
Total produced inputs used									
Primary inputs used (value-added)									
Total industry output	x								

[a] The various blocks in the table that are labeled with capital and lower-case letters are vectors and matrices that feature in later portions of the paper. They are labeled here for later referral.

which is known as an industry-occupation matrix. Across a row, an industry-occupation table shows the employment in a given occupation for each industry. Down a column, the table shows the employment in each occupational classification pertinent to a given industry. Summing across a row provides the total national employment in the given occupation. Summing down a column provides the total employment in the given industry.

The principal source for historical data on occupational employment by industry is the U.S. Bureau of Labor Statistics (BLS). Until recently this data was based principally upon the decennial U.S. Census of Population (18). Since the early 1970's, however, data on occupational employment has been obtained as part of the cooperative Federal-State program on Occupational Employment Statistics (OES), which is coordinated by the BLS (19). Figure 3 shows the published industry-occupation matrices. An unpublished matrix based on OES data was used by the BLS in their economic projections to 1990 (17).

Further Definitions and Conventions. With the background just provided, it is possible to make more explicit definitions of input-output and industry-occupation matrices that can then be used to demonstrate the usefulness of the data for analysis of occupational employment. The following definitions are used in the remainder of the paper*:

U is an n-by-n commodity-by-industry intermediate use matrix. Each element u_{ij} is the dollar value of commodity i used by industry j. The matrix is that portion of the use table (see Figure 2) which shows intermediate use of commodities by industry.

V is an n-by-n industry-by-commodity make matrix. Each element v_{ij} is the dollar value of commodity j that is produced by industry i.

y is an n-by-1 column vector of final demand. Each element y_j is the dollar value of total sales to final users for commodity j. The final demand vector is shown in Figure 2.

x is an n-by-1 column vector of industry output. Each element x_j is the dollar value of output for industry j. Data used for the industry output vector is shown in Figure 2.

x^c is an n-by-1 column vector of commodity output. Each element x_j^c is the dollar value of output of commodity j. The commodity output vector is shown in Figure 2.

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* The majority of the definitions used to develop equations (1) through (10) in this section are adapted from: Philip M. Ritz, Definitions and Conventions of the 1972 Input-Output Study, U.S. Department of Commerce, Bureau of Economic Analysis Staff Paper 34 (Washington, D.C.: U.S. Government Printing Office, 1980). The principal difference is that the present development ignores complications arising from each industry's production of scrap.

Figure 3: Industry-Occupation Tables for the United States[a]

Name	Published Source	Primary Data Source	Occupational Classification	Industry Classification
BLS 1960 national industry-occupation matrix	BLS Bulletin 1606 (20)	1960 U.S. Census	1960 Census-based system of classifying 157 detailed occupations	116 industries classified on the basis of the 1957 SIC
BLS 1970 national industry-occupation matrix; with an update for 1978	BLS Bulletin 2086 (18)	1970 U.S. Census	1970 Census-based system of classifying 377 detailed occupations	201 industries classified on the basis of the 1967 SIC
BLS 1978 national industry-occupation matrix	Publication pending; see BLS Bulletin 2112 (17)	Occupational Employment Statistics Survey	OES matrix-based system of classifying 1678 occupations	378 industries classified on the basis of the 1972 SIC

[a]Abbreviations:
BLS: U.S. Department of Labor, Bureau of Labor Statistics
OES: Occupational Employment Statistics
SIC: Standard Industrial Classification (11)

z is an n-by-1 column vector of intermediate commodity output. Each element z_j is the dollar value of commodity j used by purchasing industries for the production of their own output. The intermediate commodity output vector is shown in Figure 2.

A is an n-by-n commodity-by-industry direct requirements matrix. Each element a_{ij} is the dollar value of commodity i used by industry j to produce a dollar of that industry's output. This matrix is formed from U and x as: $a_{ij} = u_{ij}/x_j$.

D is an n-by-n industry-by-commodity market share matrix. Each element d_{ij} is the proportion of total output for commodity j produced by industry i. This matrix is formed from V and x^c as: $d_{ij} = v_{ij}/x^c_j$.

Matrices are always denoted with capital letters in the sections to follow, while lower-case letters are used for vectors. No scalars appear in the following material. Upon occasion it will also be necessary to diagonalize a vector by creating a square matrix with the values of the vector on the main diagonal and zeroes entered elsewhere. This will be indicated with a vector shown in brackets, so that

$$\{y\} = \begin{pmatrix} y_1 & 0 & . & 0 \\ 0 & y_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & y_n \end{pmatrix}$$

Use of a vector containing only 1's (i.e., the summation vector) will also be used and is denoted by the vector i. The transpose of a vector or matrix is denoted by an apostrophe; e.g., y'. Finally, row r of a given matrix is denoted by a subscript. Thus, q_r is the vector formed from row r of the Q matrix. A vector formed from column r of a matrix is denoted with a superscript, so that q^r is a vector formed from column r of the Q matrix.

It is also necessary to make two assumptions (12):

a) Industry technology assumption. It is assumed that inputs are required in proportion to output and the proportions are the same for an industry's primary and secondary products. Thus:

$$U = A\{x\} \qquad (1)$$

This equation is, of course, true for the year in which the data was collected. For use, however, one must assume the equation holds for levels of output other than those in the original vector of industry output x. This assumption will be discussed in more detail later.

b) Market share assumption. It is necessary to assume that each commodity is produced by the various industries in fixed proportions. Thus

$$V = D\{x^c\} \qquad (2)$$

Equations (3) through (10), which follow, are a derivation of a very useful matrix called the total requirements matrix. It is this matrix which can be used in conjunction with the industry-occupation matrix to analyze the effects of final demand upon occupational employment. It is a particularly important matrix because the elements of the matrix show the total effect upon a given industry resulting from delivery to final demand of a particular commodity. The important word here is "total," because this matrix shows the sum of direct effects upon an industry and the indirect effects which occur because of the interrelationships among industries in the economy.

For example, a change in the export of chemicals has a direct effect upon the output required of the chemical products industry. However, the industry is also affect indirectly through changes in the final demand of other industries. If delivery of grain products to final demand were to change, for example, this would have an indirect effect upon the chemical products industry through the sale of such commodities as fertilizers. The total requirements matrix is quite useful in that each element in the matrix shows the sum of direct and indirect effects upon a given industry resulting from the delivery to final demand of a unit of a particular commodity. This matrix will be given more specific definition later in this section.

With this as background, the derivation of the total requirements matrix is straightforward. Total commodity output is the sum of output used for intermediate demand and for final demand:

$$x^c = z + y \tag{3}$$

Intermediate commodity output, z, is the product of the direct requirements matrix and the vector of industry output:

$$z = Ax \tag{4}$$

Substitution of equation (4) into equation (3) yields:

$$x^c = Ax + y \tag{5}$$

Industry output, x, is just the product of the market share matrix and the vector of commodity output:

$$x = Dx^c \tag{6}$$

Substituting (6) into (5) and solving for x^c gives:

$$x^c = ADx^c + y \tag{7}$$

$$(I - AD)x^c = y \tag{8}$$

$$x^c = (I - AD)^{-1}y \tag{9}$$

Substitution of equation (9) into equation (6) gives:

$$x = D(I - AD)^{-1}y \tag{10}$$

In equation (10) the matrix $D(I - AD)^{-1}$ is the total requirements matrix, relating final demand to industry output. If we represent this matrix as Q, then a more formal definition of the matrix is given as:

> Q is an n-by-n industry-by-commodity matrix where each element q_{ij} is the dollar value of output required both directly and indirectly from industry i per dollar of delivery to final demand of commodity j. The column total is the cumulative effect on the economy of a dollar increase in final demand for a given commodity. This is often referred to as the output multiplier.

Two final definitions are needed before it is possible to proceed to the presentation of mathematical expressions relating final demand to occupational employment:

> E is a k-by-n occupation-by-industry matrix of employment. Each element e_{ij} is the employment in occupation i in industry j.

> L is a k-by-n occupation-by-industry matrix of labor input ratios. Each element l_{ij} is the ratio of employment in occupation i in industry j to the total output of industry j. Each element in L is formed as: $l_{ij} = e_{ij}/x_j$.

Expressions for the Interindustry Analysis of Employment. A series of useful formulas can be developed which relate final demand to industry and occupational employment. In this section the expressions will be shown in unit terms, meaning that the result that occurs from the various manipulations of vectors and matrices are expresed in terms of units of final demand. How actual patterns of final demand can be used in conjunction with the unit terms constructed here will be delayed until the final section on applications. In what remains of this section the formulas are presented and discussed individually. The next section uses a simple example to work through the formulas in an effort to make their meaning clear.

a) Unit matrix of employment by industry and commodity: $\{L'i\}Q$
Each element ij in this matrix is the total employment in industry i required per dollar of commodity j delivered to final demand.

b) Unit vector of employment by commodity: $i'\{L'i\}Q$
Each element j in this vector is the total employment per dollar of delivery to final demand of commodity j.

c) Unit matrix of employment by occupation and commodity: LQ
Each element ij gives the employment in occupation i per dollar of delivery to final demand of commodity j.

d) Unit occupational profile matrix: $\{l_r\}Q$, where l_r is row r of L
Each element ij is the employment in occupation r required in industry i per dollar of delivery to final demand of commodity j.

e) Unit industry profile matrix: $l^r q_r$, where l^r is column r of L and q_r is row r of Q.
Each element ij is the employment in occupation i required in

industry r per dollar of delivery to final demand of commodity j.

f) Unit commodity profile matrix: $L\{q^r\}$, where q^r is column r in Q. Each element ij is the employment in occupation i required by industry j per dollar of delivery to final demand of commodity r.

An Example. To make clear how the various unit matrices and vectors are constructed, it is useful to work through a simple example. In Figure 4, a use table is shown for an economy with three industries and three commodities. The make table is shown in Figure 5. Figure 6 shows employment by occupation and industry. Note that the industries are labeled A, B, and C. The commodities are labeled a, b, and c, with commodity 'a' being primary to industry A, 'b' being primary to industry B, and commodity c being primary to industry C. There are two occupations in the economy, labeled as occupation 1 and occupation 2.

With the data in Figures 4 through 6 it is possible to construct all of the unit matrices and vectors described in the previous section. This proceeds below in a series of steps.

Step 1: Matrix of direct requirements: A

Each element $a_{ij} = u_{ij}/x_j$. The result of these computations is:

	Industry		
Commodity	A	B	C
a	.143	.028	.132
b	.429	.056	.105
c	.286	.444	.211

Step 2: Market share matrix: D

Each element $d_{ij} = v_{ij}/x_j^c$. The result is shown below:

	Commodity		
Industry	a	b	c
A	.750	.050	0
B	.250	.600	.250
C	0	.350	.750

Step 3: Total requirements matrix: Q

The total requirements matrix is constructed as $Q = D(I - AD)^{-1}$. The series of computations leading to this result is shown below.

i) AD: $\begin{pmatrix} .114 & .070 & .106 \\ .336 & .092 & .093 \\ .326 & .355 & .269 \end{pmatrix}$

Figure 4: The Use of Commodities by Industries (in millions of dollars at producers' prices)

Commodity	Producing Industry A	B	C	Total Intermediate Demand	Final Demand	Commodity Output
a	50	25	125	200	200	400
b	150	50	100	300	700	1000
c	100	400	200	700	100	800
Value-added	50	425	525			
Industry Output	350	900	950			

Figure 5: The Make of Commodities by Industries (in millions of dollars at producers' prices)

Industry	Commodity a	b	c	Industry Output
A	300	50	0	350
B	100	600	200	900
C	0	350	600	950
Commodity Output	400	1000	800	

Figure 6: Employment by Industry and Occupation (in thousands)

Occupation	Industry A	B	C	Totals
1	2	6	4	12
2	8	4	1	13
Totals	10	10	5	25

ii) $(I - AD)$: $\begin{pmatrix} .886 & -.070 & -.106 \\ -.336 & .908 & -.093 \\ -.326 & -.355 & .731 \end{pmatrix}$

iii) $(I - AD)^{-1}$: $\begin{pmatrix} 1.2729 & .1792 & .2074 \\ .5569 & 1.2374 & .2382 \\ .8381 & .6808 & 1.5761 \end{pmatrix}$

iv) $Q = D(I - AD)^{-1}$: $\begin{pmatrix} .983 & .196 & .167 \\ .862 & .957 & .589 \\ .823 & .944 & 1.265 \end{pmatrix}$

Step 4: Labor input matrix: L

Each element $l_{ij} = e_{ij}/x_j$. Here and in the remainder of this example the units will be per millions of dollars of output. The results of the computations are:

	Industry		
Occupation	A	B	C
1	5.714	6.667	4.211
2	22.857	4.444	1.053
Totals	28.571	11.111	5.263

Step 5: Unit matrix of employment by industry and commodity: $\{L'i\}Q$

i) $L'i$: $\begin{pmatrix} 5.714 & 22.857 \\ 6.667 & 4.444 \\ 4.211 & 1.053 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 28.571 \\ 11.111 \\ 5.263 \end{pmatrix}$

ii) $\{L'i\}Q$: $\begin{pmatrix} 28.571 & 0 & 0 \\ 0 & 11.111 & 0 \\ 0 & 0 & 5.263 \end{pmatrix} \begin{pmatrix} .983 & .196 & .167 \\ .862 & .957 & .589 \\ .823 & .944 & 1.265 \end{pmatrix}$

$= \begin{pmatrix} 28.085 & 5.600 & 4.771 \\ 9.578 & 10.633 & 6.544 \\ 4.331 & 4.968 & 6.658 \end{pmatrix}$

Each element ij in $\{L'i\}Q$ is the total employment in industry i required per million dollars of commodity j delivered to final demand. For example, the element in the (1,1) cell, 28.085, is the total employment in Industry A required per million dollars of Commodity a delivered to final demand. Stated differently, when $1 million of Commodity a is delivered to final demand, the employment generated in Industry A is 28,085. It should be clear that

the $1 million of Commodity a can be made by any industry. In otherwords, the cell should not be interpreted as the employment in Industry A required when Industry A delivers $1 million of Commodity a to final demand. This same observation holds for all of the other unit terms presented earlier and shown by example in later steps in this section.

Step 6: Unit vector of employment by commodity: $i'\{L'i\}Q$

$$i'\{L'i\}Q = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}28.085 & 5.600 & 4.771\\ 9.578 & 10.633 & 6.544\\ 4.331 & 4.968 & 6.658\end{pmatrix}$$

$$= \begin{pmatrix}41.994 & 21.201 & 17.973\end{pmatrix}$$

Each element j gives the total employment per million dollars of delivery to final demand of commodity j. For example, the element in the second cell, 21.201, is the total employment (in thousands) when $1 million of Commodity b is delivered to final demand.

Step 7: Unit matrix of employment by occupation and commodity: LQ

$$LQ = \begin{pmatrix}5.714 & 6.667 & 4.211\\ 22.857 & 4.444 & 1.053\end{pmatrix}\begin{pmatrix}.983 & .196 & .167\\ .862 & .957 & .589\\ .823 & .944 & 1.265\end{pmatrix}$$

$$= \begin{pmatrix}14.829 & 11.475 & 10.208\\ 27.166 & 9.727 & 7.767\end{pmatrix}$$

Each element ij gives the employment in occupation i per delivery to final demand of $1 million of commodity j. For example, the element in cell (2,3), 7.767, is the employment in Occupation 2 required per delivery to final demand of $1 million of Commodity c. When $1 million of Commodity c is delivered to final demand, the employment generated in Occupation 2 is 7767.

Step 8: Unit occupational profile matrix: $\{1_r\}Q$

For Occupation 1 (row 1 in L),

$$\{1_1\} = \begin{pmatrix}5.714 & 0 & 0\\ 0 & 6.667 & 0\\ 0 & 0 & 4.211\end{pmatrix}$$

$$\{1_1\}Q = \begin{pmatrix}5.617 & 1.120 & .954\\ 5.747 & 6.380 & 3.927\\ 3.466 & 3.975 & 5.327\end{pmatrix}$$

Each element ij gives employment in Occupation 1 in industry i per delivery of $1 million of commodity j to final demand. For example, the element in cell (2,3), 3.927, is the employment in Occupation 1 required in Industry B per delivery to final demand of $1 million of Commodity c. When $1 million of Commodity c is delivered to final demand, the employment

in Occupation 1 generated in Industry B is 3927.

Step 9: Unit industry profile matrix: $1_r q^r$

For Industry B (i.e., column 2 of L and row 2 of Q), the computations are:

$$1^2 q_2 = \begin{pmatrix} 6.667 \\ 4.444 \end{pmatrix} \begin{pmatrix} .862 & .957 & .589 \end{pmatrix}$$

$$= \begin{pmatrix} 5.747 & 6.380 & 3.927 \\ 3.831 & 4.253 & 2.618 \end{pmatrix}$$

Each element ij is the employment in occupation i required per delivery to final demand of $1 million of commodity j, for Industry B. For example, the element in cell (2,1), 3.831, is the employment in Occupation 2 required in Industry B per delivery to final demand of $1 million of Commodity a. When $1 million of Commodity a is delivered to final demand, the employment in Occupation 2 generated in Industry B is 3831.

Step 10: Unit commodity profile matrix: $L\{q^r\}$

For Commodity a (i.e., column 1 of Q), the computations are:

$$L\{q^1\} = \begin{pmatrix} 5.714 & 6.667 & 4.211 \\ 22.857 & 4.444 & 1.053 \end{pmatrix} \begin{pmatrix} .983 & 0 & 0 \\ 0 & .862 & 0 \\ 0 & 0 & .823 \end{pmatrix}$$

$$= \begin{pmatrix} 5.617 & 5.747 & 3.466 \\ 22.468 & 3.831 & .867 \end{pmatrix}$$

Each element ij is the employment in occupation i required by industry j per delivery of $1 million of Commodity a to final demand. For example, the element in cell (1,3), 3.466, is the employment required in Occupation 1 in Industry C per delivery to final demand of $1 million of Commodity a. When $1 million of Commodity a is delivered to final demand, the employment in Occupation 1 generated in Industry C is 3466.

It should be clear that each of the unit profile matrices, for occupations, industries, or commodities, presents the same information rearranged according to use. The matrices are particularly useful when one is interested primarily in only one occupation, industry, or commodity. For example, the unit commodity profile matrix and the unit industry profile matrix can be used in the analysis of employment by a single industry.

Applications. Each of the unit matrices and vectors just developed expresses a relationship between industry or occupational employment and final demand. Each effect upon industry or occupational employment was stated per unit of final demand (i.e., per dollar or per million dollars). These unit vectors and matrices find use when combined in different ways with specific patterns of final demand. In this section some of these uses will be identified and then illustrated with the same example used in the previous section.

Figures 7 and 8 show a variety of uses of the unit terms with patterns of

Figure 7: Use of Unit Vectors and Matrices Relating Industry and Occupational Employment to Specific Patterns of Final Demand

Unit Matrix or Vector	Final Demand y	Final Demand {y}
{L'i}Q : unit matrix of employment by industry and commodity	Cell 1: Employment by industry	Cell 2: Employment by industry and commodity
i'{L'i}Q: unit vector of employment by commodity	Cell 3: Total national employment	Cell 4: Employment by commodity
LQ: unit matrix of employment by occupation and commodity	Cell 5: Employment by occupation	Cell 6: Employment by occupation and commodity
$\{1_r\}Q$: unit occupational profile matrix	Cell 7: Employment by industry, for a specific occupation	Cell 8: Employment by industry and commodity for a specific occupation
$1^r q_r$: unit industry profile matrix	Cell 9: Employment by occupation, for a given industry	Cell 10: Employment by occupation and commodity, for a given industry

Figure 8: Uses of the Unit Commodity Profile Matrix*

Unit term	Final Demand y^r	Final Demand $\{y^r\}$
$L\{q^r\}$: unit commodity profile matrix	Cell A: Employment by occupation, for a given commodity	Cell B: Employment by occupation and industry, for a given commodity

* y^r is element r of y times the n-by-1 vector i .

final demand. Each unit matrix or vector can be right-multiplied by a column vector of fianl demand or a diagonalized matrix where the elements of final demand are on the principal diagonal and all other matrix elements are zero. Each of these produces a different result, as shown in the various cells of Figures 7 and 8. In addition, the traditional type alpha manpower matrix showing employment by occupation and industry can be simply generated as L{Qy}.

The same example presented in an earlier section will be used to show the application of the unit matrices and vectors. While any vector of final demand can be used with the unit terms, the example to follow uses the original vector of final demand shown in the use table of Figure 4. This is done because many of the computations involving this final demand vector and the unit matrices yield results which are clearly related to the original input-output and industry-occupation data in Figures 4 through 6. This is done only for didactic purposes here. An actual application of the unit terms would use a final demand vector which was different from that used to construct the unit terms.

Cell 1: Employment by industry: {L'i}Qy

$$\{L'i\}Qy = \begin{pmatrix} 28.085 & 5.600 & 4.771 \\ 9.578 & 10.633 & 6.544 \\ 4.331 & 4.968 & 6.658 \end{pmatrix} \begin{pmatrix} 200 \\ 700 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} 10,014 \\ 10,013 \\ 5,009 \end{pmatrix}$$

Each element j is the total employment in industry j required to deliver to final demand the amounts specified in y. Except for rounding errors, the resultant vector in this example is just employment by industry shown in Figure 6.

Cell 2: Employment by industry and commodity: {L'i}Q{y}

$$\{L'i\}Q\{y\} = \begin{pmatrix} 28.085 & 5.600 & 4.771 \\ 9.578 & 10.633 & 6.544 \\ 4.331 & 4.968 & 6.658 \end{pmatrix} \begin{pmatrix} 200 & 0 & 0 \\ 0 & 700 & 0 \\ 0 & 0 & 100 \end{pmatrix}$$

$$= \begin{pmatrix} 5617 & 3920 & 477 \\ 1916 & 7464 & 654 \\ 866 & 3478 & 666 \end{pmatrix}$$

Each element ij is the total employment in industry i required to deliver to final demand the commodity j in the amount specified by y. Standard column and row comparisons can be used to produce useful information in this and other results to follow. For example, it is interesting that even though Commodity c is the primary output of Industry C, the employment in Industry C is influenced most substantially by Commodity b (e.g., compare cell (3,2) with cell (3,3)). This occurs because the level of final demand for Commodity b is seven times the level of Commodity c delivered to final demand.

Cell 3: Total national employment: i'{L'i}Qy

$$i'\{L'i\}Qy = (41.994 \quad 21.201 \quad 17.973)\begin{pmatrix}200\\700\\100\end{pmatrix}$$

$$= 25{,}037$$

Except for errors caused by rounding, this result is the total of national employment shown in Figure 6.

Cell 4: Employment by commodity: $i'\{L'i\}Q\{y\}$

$$i'\{L'i\}Q\{y\} = (41.994 \quad 21.201 \quad 17.973)\begin{pmatrix}200 & 0 & 0\\0 & 700 & 0\\0 & 0 & 100\end{pmatrix}$$

$$= (8399 \quad 14{,}841 \quad 1797)$$

Each element j is the total employment required to deliver Commodity j to final demand in the amount specified by y. While Commodity a accounts for 20% of total final demand, one-third of the total national employment occurs as a result of the delivery to final demand of this commodity (i.e., 8399/25,000).

Cell 5: Employment by occupation: LQy

$$LQy = \begin{pmatrix}14.829 & 11.475 & 10.208\\27.166 & 9.727 & 7.767\end{pmatrix}\begin{pmatrix}200\\700\\100\end{pmatrix}$$

$$= \begin{pmatrix}12{,}019\\13{,}019\end{pmatrix}$$

Each element j is the total employment in occupation j required to deliver to final demand the amounts of commodities specified by y. Except for errors of rounding, the results are those shown in Figure 6 for total occupational employment.

Cell 6: Employment by occupation and commodity: $LQ\{y\}$

$$LQ\{y\} = \begin{pmatrix}14.829 & 11.475 & 10.208\\27.166 & 9.727 & 7.767\end{pmatrix}\begin{pmatrix}200 & 0 & 0\\0 & 700 & 0\\0 & 0 & 100\end{pmatrix}$$

$$= \begin{pmatrix}2966 & 8033 & 1021\\5433 & 6809 & 777\end{pmatrix}$$

Each element ij is the total employment in occupation i required to deliver to final demand the commodity j in the amount specified by y. From this matrix we learn that, for the given pattern of final demand, Occupation 1 is most heavily influenced by Commodity b, while Occupation 2 is more evenly influenced by Commodities a and b. Approximately two-thirds of the total employment in

Occupation 1 (8033) is generated by delivery to final demand of $700,000,000 of Commodity b.

Cell 7: Employment by industry, for a given occupation: $\{1^r\}Qy$

$$\{1^1\}Qy = \begin{pmatrix} 5.617 & 1.120 & .954 \\ 5.747 & 6.380 & 3.927 \\ 3.466 & 3.975 & 5.327 \end{pmatrix} \begin{pmatrix} 200 \\ 700 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} 2003 \\ 6008 \\ 4008 \end{pmatrix}$$

Each element j is the employment in occupation r required in industry j when commodities in the amount specified by y are delivered to final demand. Except for rounding errors, the results shown above for Occupation 1 are the same as the values shown in Figure 6.

Cell 8: Employment by industry and commodity, for a given occupation: $\{1^r\}Q\{y\}$

$$\{1^1\}Q\{y\} = \begin{pmatrix} 5.617 & 1.120 & .954 \\ 5.747 & 6.380 & 3.927 \\ 3.466 & 3.975 & 5.327 \end{pmatrix} \begin{pmatrix} 200 & 0 & 0 \\ 0 & 700 & 0 \\ 0 & 0 & 100 \end{pmatrix}$$

$$= \begin{pmatrix} 1123 & 784 & 95 \\ 1149 & 4466 & 393 \\ 693 & 2783 & 533 \end{pmatrix}$$

Each element ij is the employment in occupation r required in industry i to deliver commodity j to final demand in the amount specified by y. The matrix shows, for example, that nearly 70% of the employment in Occupation 1 in Industry C (i.e., 2783/4008) is generated by the sale of $700 million of Commodity b to final users.

Cell 9: Employment by occupation, for a given industry: $1^r q_r y$

$$1^3 q_3 y = \begin{pmatrix} 3.466 & 3.975 & 5.327 \\ .867 & .994 & 1.332 \end{pmatrix} \begin{pmatrix} 200 \\ 700 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} 4008 \\ 1002 \end{pmatrix}$$

Each element j is the employment in occupation j required in industry r to deliver to final demand commodities in the amounts specified by y. In Industry C, the total employment in Occupation 1 generated by the specified pattern of expenditures on final demand is four times the employment generated in Occupation 2.

Cell 10: Employment by occupation and commodity, for a given industry: $1^r q_r \{y\}$

Again using Industry C as the example, the computations are:

$$1^{3}q_{3}\{y\} = \begin{pmatrix} 3.466 & 3.975 & 5.327 \\ .867 & .994 & 1.332 \end{pmatrix} \begin{pmatrix} 200 & 0 & 0 \\ 0 & 700 & 0 \\ 0 & 0 & 100 \end{pmatrix}$$

$$= \begin{pmatrix} 693 & 2783 & 533 \\ 173 & 696 & 133 \end{pmatrix}$$

Each element ij is the employment in occupation i required in industry r to deliver to final demand the commodity amounts specified by y. Seventeen percent of the employment in Occupation 2 in Industry C is generated as the result of the delivery to final demand of $200 million of Commodity a.

Cell A: Employment by occupation, for a given commodity: $L\{q^{r}\}y^{r}$

$$L\{q^{1}\}y^{1} = \begin{pmatrix} 5.617 & 5.747 & 3.466 \\ 22.468 & 3.831 & .867 \end{pmatrix} \begin{pmatrix} 200 \\ 200 \\ 200 \end{pmatrix}$$

$$= \begin{pmatrix} 2966 \\ 5433 \end{pmatrix}$$

Each element j is the total employment in occupation j required to deliver commodity r to final demand in the amount specified by y. For example, the delivery to final demand of $200 million of Commodity a requires employment in Occupation 1 of 2966 and employment in Occupation 2 of 5433.

Cell B: Employment by occupation and industry, for a given commodity: $L\{q^{r}\}\{y^{r}\}$

$$L\{q^{1}\}\{y^{1}\} = \begin{pmatrix} 5.617 & 5.747 & 3.466 \\ 22.468 & 3.831 & .867 \end{pmatrix} \begin{pmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{pmatrix}$$

$$= \begin{pmatrix} 1123 & 1149 & 693 \\ 4494 & 766 & 173 \end{pmatrix}$$

Each element ij is the employment in occupation i in industry j required to deliver commodity r to final demand in the amount specified in y. This provides a breakdown of the results shown in Cell A. For example, of the employment required in Occupation 2 (5433) to deliver $200 million of Commodity a to final demand, nearly 83% is required by Industry A (i.e., 4494).

Type alpha manpower matrix: Employment by occupation and industry: $L\{Qy\}$

$$Qy = \begin{pmatrix} .983 & .196 & .167 \\ .862 & .957 & .589 \\ .823 & .944 & 1.265 \end{pmatrix} \begin{pmatrix} 200 \\ 700 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} 350.5 \\ 901.2 \\ 951.9 \end{pmatrix}$$

$$L\{Qy\} = \begin{pmatrix} 5.714 & 6.667 & 4.211 \\ 22.857 & 4.444 & 1.053 \end{pmatrix} \begin{pmatrix} 350.5 & 0 & 0 \\ 0 & 901.2 & 0 \\ 0 & 0 & 951.9 \end{pmatrix}$$

$$= \begin{pmatrix} 2003 & 6008 & 4008 \\ 8011 & 4005 & 1002 \end{pmatrix}$$

Each element ij is the employment in occupation i in industry j required to deliver to final demand the commodity amounts specified by y. Except for errors of rounding the results shown above are those displayed earlier in Figure 6.

The utility of the unit matrices and vectors is greatest when used to perform "what if" simulations. In the previous example, the vector of final demand shown in the original use table was multiplied by the unit terms. This was done for purposes of clarity, so that it would be obvious what the matrix manipulations were producing. However, any vector of final demand could have been used in the ways shown in Figures 7 and 8. When this is done, one engages in simulations in which the effect of a particular pattern of final demand upon industry and occupational employment is assessed.

There is, for example, considerable speculation at present regarding the effect of "high technology" growth on employment. Assuming that high technology industries and commodities could be identified, the analyses shown in Figures 7 and 8 could be performed to provide a comprehensive review of the effects upon industry and occupational employment when high technology industries grow by a specified amount. This growth would be operationalized by specifying a vector of final demand for the commodities produced in the economy. We could, for example, address the following type of question: What is the effect upon employment when "high tech" commodities grow by 10% while other commodities remain at their present levels? Using the procedures shown in Figures 7 and 8, answers to this question could deal with employment by occupation, by industry, by commodity, or by some combination of these. The various profile matrices could by used to isolate the effect on a specific occupation or industry, or the effects due to a specific commodity.

The usefulness of this type of "what if" simulation, however, is limited by the assumptions made in operationalizing procedures shown in Figures 7 and 8. Recall that three principal assumptions were made:

1. It must be assumed that each commodity is produced by the various industries in fixed proportions.
2. It must be assumed that industry technology does not change, meaning that inputs are required in fixed proportions to the output of the industry.
3. It must be assumed that occupational employment by industry does not change, meaning that occupational staffing ratios within an industry are constant.

These three assumptions restrict the usefulness of the interindustry analysis of employment by occupation and industry. In terms of the equations, the first two assumptions mean that the total requirements matrix, Q, remains constant over time. Across short periods of time and within limited ranges of final demand, this may be a reasonable assumption. But input-output studies have shown that the total requirements matrices do change (22), even from year to

year. The assumption of constant staffing patterns in industries is also not consistent with observation. It is known, for example, that occupational staffing patterns change over time with changes in technology, with a shifting product mix of the industry, with changes in relative prices, and simply because there is a non-proportional relationship between output and the requirements for certain types of labor (10,20). These same factors are also responsible for changes in the total requirements matrix.

Consequently, "what if" simulations of the kind just discussed are limited by the assumptions necessary. If one is dealing with incremental changes in final demand, over restricted periodsof time, then these simulations may provide reasonable indications of the effects upon industry and occupational employment. However, one can relax the assumptions and attain a more general simulation model. To the extent that one is able to translate various scenarios into their effects upon Q and L, the analyses in Figures 7 and 8 can be used in a broad simulation model.

Continuing with the high technology example, one could assess the effect upon industry and occupational employment under the following scenario:

1. Assume that the final demand of "high tech" commodities increases by 10% while that for all other commodities remains constant.
2. Assume that employment in high technology industries is deskilled, so that occupational staffing patterns in these industries change in such a way that low skill occupations comprise 10% more of the total employment in the industries. Use this assumption to change L, the matrix of occupation-by-industry labor input ratios.
3. Assume that there is a general increase in the capital intensity of all industries, as automation becomes more prevalant. Translate this into changes in the direct requirements matrix, A, and from here into Q. For example, we might increase the total input of capital equipment commodities in A by a uniform 5% across all industries.

The scenario just constructed could then be assessed for its effect on industry and occupational employment using the equations in Figures 7 and 8. A variety of other scenarios can also be constructed and are limited only by one's ability to translate situations into their effects upon the component terms in the equations. These scenarios are not intended to be predictions of what will happen. They are simply explorations of what would happen if certain conditions occurred. The utility of such simulation exercises lies in their ability to guide policy. The simulations don't identify policies to follow. They don't tell one how to achieve a certain outcome. But they do show what would happen if certain conditions occur. Depending upon the individual decision-maker's value orientation, simulation results can be suggestive of policies that are needed to avoid the scenario or to improve the chances of its occurring.

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